PIPES AND CISTERNS

In previous classes, we have studied about the concepts of time and work. Pipes and cisterns are related to time and work. Pipes and cisterns problems are asked in various competitive exams.

To solve the problems of pipes and cisterns, we should understand the terms given below.

Inlet: An inlet is a pipe which is connected with a tank or a cistern or a reservior which fills it.

Outlet or Leak: An outlet is a pipe which is connected with a tank or a cistern or a reservior which empties it.

Basic Concepts and Formulae

(*i*) If an inlet pipe can fill a tank in x hours, then part of the tank filled by pipe in 1 hour $= \frac{1}{x}$.

(*ii*) If an outlet pipe can empty a tank in y hours, then part of the tank emptied by pipe in 1 hour $= \frac{1}{y}$.

(*iii*) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours, then on opening both the pipes together:

Case I. When $y > x$.

The part of the tank filled in 1 hour $= \frac{1}{x} - \frac{1}{y}$.

The time taken to fill the tank $= \frac{xy}{y-x}$.

Case II. When $x > y$.

The part of the tank emptied in 1 hour $= \frac{1}{y} - \frac{1}{x}$.

The time taken to empty the tank $= \frac{xy}{x-y}$.

(*iv*) If a pipe can fill the tank in x hours and another pipe can fill the same tank in y hours, then on opening both the pipes together:

The part of the tank filled in 1 hour $= \frac{1}{x} + \frac{1}{y}$.

The time taken to fill the tank $= \frac{xy}{x+y}$.

(*v*) If a pipe can fill the tank in x hours and another pipe can fill the same tank in y hours but a third pipe can empty the full tank in z hours, then on opening all the pipes together:

The part of the tank filled in 1 hour $= \frac{1}{x} + \frac{1}{y} - \frac{1}{z}$.

The time taken to fill the tank $= \frac{xyz}{yz + zx - xy}$.

ILLUSTRATIVE EXAMPLES

Example 1. *A pump can fill a tank with water in 20 minutes and another pump can fill the same tank in 30 minutes. If both the pumps are opened together, then how much time will be taken to fill the tank completely?*

Solution. Part of the tank filled by first pump in 1 minute $= \frac{1}{20}$.

Part of the tank filled by second pump in 1 minute $= \frac{1}{30}$.

Part of the tank filled by both the pumps in 1 minute

$$= \frac{1}{20} + \frac{1}{30} = \frac{3+2}{60} = \frac{1}{12}.$$

$\therefore$ The time taken to fill the tank = 12 minutes.

Example 2. *A pipe can fill the tank in 3 hours and another pipe can empty the full tank in 4 hours. If both the pipes are opened together, then find how much time will they take to fill the tank?*

Solution. Part of the tank filled by first pipe in 1 hour $= \frac{1}{3}$.

Part of the tank emptied by second pipe in 1 hour $= \frac{1}{4}$.

Part of the tank filled by both the pipes in 1 hour $= \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$.

$\therefore$ The time taken to fill the tank = 12 hours.

Example 3. *A pump can fill a tank with water in 3 hours because of a leakage in the tank it took* $3\frac{1}{3}$ *hours to fill the tank. How much time will it take for the leakage to drain all the water of the full tank?*

Solution. Let leakage in the tank take x hours to drain the full tank, then

part of the tank emptied by the leakage in 1 hour $= \frac{1}{x}$.

Part of the tank filled by the pump in 1 hour $= \frac{1}{3}$.

When both pump and leakage are working, then since due to leakage it took $3\frac{1}{3}$ hours *i.e.* $\frac{10}{3}$ hours to fill the tank, so part of the tank filled in 1 hour $= \frac{3}{10}$ (given).

$\therefore \quad \frac{1}{3}-\frac{1}{x}=\frac{3}{10} \Rightarrow \frac{1}{x}=\frac{1}{3}-\frac{3}{10}$

$\Rightarrow \quad \frac{1}{x}=\frac{1}{30} \Rightarrow x=30$

$\therefore$ The leakage will take 30 hours to drain the full tank.

Example 4. *Two pipes A and B together can fill a tank in 10 minutes. If pipe A takes 15 minutes less than B to fill the tank alone, then find the time taken by pipe B to fill the tank alone.*

Solution. Let pipe B fill the tank in x minutes, then pipe A will fill the tank in $(x-15)$ minutes.

$\therefore \quad \frac{1}{x}+\frac{1}{x-15}=\frac{1}{10}$

$\Rightarrow \quad \frac{2x-15}{x(x-15)}=\frac{1}{10} \Rightarrow x^2-15x=20x-150$

$\Rightarrow \quad x^2-35x+150=0$

$\Rightarrow \quad (x-30)(x+5)=0$

$\Rightarrow \quad x=30 \qquad (\because x \neq -5)$

Hence, B alone will fill the tank in 30 minutes.

Example 5. *Pipe A can fill the tank 2 times faster than pipe B. If both pipes A and B running together can fill the tank in 24 minutes, find how much time will pipe B alone take to fill the tank?*

Solution. Let B alone fill the tank in x minutes, then A alone will fill the tank in $\frac{x}{2}$ minutes.

$\therefore \quad \frac{1}{x}+\frac{2}{x}=\frac{1}{24} \Rightarrow \frac{3}{x}=\frac{1}{24}$

$\Rightarrow \quad x=72$

Hence, B alone will fill the tank in 72 minutes.

Example 6. *Pipes A and B can fill a tank in 6 hours and 8 hours respectively. Pipe C can empty it in 12 hours. If all the three pipes are opened together, find the time in which the tank can be filled.*

Solution. Part of the tank filled by pipe A in 1 hour $= \frac{1}{6}$.

Part of the tank filled by pipe B in 1 hour $= \frac{1}{8}$.

Part of the tank emptied by pipe C in 1 hour $= \frac{1}{12}$.

$\therefore$ Part of the tank filled by A, B and C in 1 hour $= \frac{1}{6}+\frac{1}{8}-\frac{1}{12}$

$= \frac{4+3-2}{24}=\frac{5}{24}$.

$\therefore$ Time taken to fill the tank $= \frac{24}{5}$ hours $= 4\frac{4}{5}$ hours.

Example 7. *Two pipes can fill a cistern in 8 and 12 hours respectively. The pipes are opened simultaneously and it takes 12 minutes more to fill the cistern due to leakage. If the cistern is full, what will be the time taken by the leakage to empty it?*

Solution. Part of the cistern filled by both the pipes in 1 hour $= \frac{1}{8}+\frac{1}{12}=\frac{5}{24}$.

So, time taken by both the pipes to fill the cistern $= \frac{24}{5}$ hours $= 4$ h 48 min.

Now, time taken to fill the cistern due to leakage = 4 h 48 min + 12 min = 5 hours.

So, part of the cistern filled by both the pipes and the leakage in 1 hour = $\frac{1}{5}$.

$\therefore$ Part of the cistern emptied by the leakage in 1 hour = $\frac{5}{24} - \frac{1}{5} = \frac{1}{120}$.

Hence, time taken by the leakage to empty the full cistern = 120 hours.

Example 8. *A cistern can be filled by two pipes A and B in 12 minutes and 15 minutes respectively. Another tap C can empty the full tank in 20 minutes. If the tap C is opened 5 minutes after the pipes A and B are opened, find when the cistern becomes full?*

Solution. Part of the cistern filled by the pipes A and B in one minute = $\frac{1}{12} + \frac{1}{15}$.

$\therefore$ Part of the cistern filled by the pipes A and B in 5 minutes = $5.\left(\frac{1}{12} + \frac{1}{15}\right) = 5 \times \frac{27}{180} = \frac{3}{4}$.

So, remaining part of the cistern = $1 - \frac{3}{4} = \frac{1}{4}$.

Now, part of the cistern filled by A, B and C in 1 minute = $\frac{1}{12} + \frac{1}{15} - \frac{1}{20} = \frac{18}{180} = \frac{1}{10}$.

So, $\frac{1}{4}$ part of the cistern will be filled by A, B and C = $\frac{1}{4} \times 10 = \frac{5}{2}$ minutes

= 2 minutes 30 seconds.

Hence, the cistern will becomes full in 5 minute + 2 min 30 sec. = 7 min 30 sec.

Example 9. *A cistern can be filled by an inlet pipe in 20 hours and can be emptied by an outlet pipe in 25 hours. Both the pipes are opened. After 10 hours, the outlet pipe is closed, find the total time taken to fill the cistern.*

Solution. Part of the cistern filled by the inlet pipe in 1 hour = $\frac{1}{20}$.

Part of the cistern emptied by the outlet pipe in 1 hour = $\frac{1}{25}$.

Part of the cistern filled when both the pipes are opened in 1 hour = $\frac{1}{20} - \frac{1}{25}$

Part of the cistern filled when both the pipes are opened in 10 hours = $10\left(\frac{1}{20} - \frac{1}{25}\right) = \frac{1}{10}$.

Remaining part of the cistern = $1 - \frac{1}{10} = \frac{9}{10}$.

Time taken by the inlet pipe to fill the remaining part of the tank = $\left(\frac{9}{10} \times 20\right)$ hours = 18 hours.

So, total time taken to fill the tank = (10 + 18) hours = 28 hours.

Example 10. *A tank is fitted with 3 taps A, B and C. All the three taps, if opened together can drain the full tank in* $1\frac{1}{2}$ *minutes. Taps B and C takes 2 minutes to drain the tank while A and C together take* $2\frac{4}{13}$ *minutes to drain it. How long will taps A and B together take to drain the tank?*

Solution. Let the time taken by taps A, B and C to drain the full tank alone be x minutes, y minutes and z minutes respectively, then

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{\frac{3}{2}}$$

i.e. $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{2}{3}$ ...(*i*)

$\frac{1}{y}+\frac{1}{z}=\frac{1}{2}$ ...(*ii*)

and $\frac{1}{x}+\frac{1}{z}=\frac{1}{\frac{30}{13}}$

i.e. $\frac{1}{x}+\frac{1}{z}=\frac{13}{30}$ ...(*iii*)

Operating 2 × (*i*) – (*ii*) – (*iii*), we get

$$2\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)-\left(\frac{1}{y}+\frac{1}{z}\right)-\left(\frac{1}{x}+\frac{1}{z}\right)=\frac{4}{3}-\frac{1}{2}-\frac{13}{30}$$

$$\Rightarrow \quad \frac{1}{x}+\frac{1}{y}=\frac{40-15-13}{30}=\frac{12}{30}=\frac{2}{5}.$$

$\therefore$ Time taken by taps A and B to drain the tank together = $\frac{5}{2}$ min = 2 min 30 sec.

Example 11. *Two pipes A and B can fill a tank in 30 minutes and 45 minutes respectively. Both pipes A and B are opened together for some time and then pipe B is turned off. If the tank is filled in 20 minutes, then find after how many minutes the pipe B is turned off?*

Solution. Part of the tank filled by A in 1 minute = $\frac{1}{30}$.

Part of the tank filled by B in 1 minute = $\frac{1}{45}$.

Let pipe B be turned off after t minutes, then

$$t\left(\frac{1}{30}+\frac{1}{45}\right)+(20-t)\times\frac{1}{30}=1$$

$$\Rightarrow \quad t\left(\frac{3+2}{90}\right)+\frac{60-3t}{90}=1$$

$$\Rightarrow \quad 5t+60-3t=90 \Rightarrow 2t=30$$

$$\Rightarrow \quad t=15.$$

Hence, pipe B is turned off after 15 minutes.

Example 12. *Three pipes A, B and C are installed to fill a tank. Pipes A and B opened together can fill the tank in the same time in which C can alone fill the tank. If pipe B can fill the tank 15 minutes faster than pipe A and 5 minutes slower than pipe C, then find the time required by pipe A to fill the tank alone.*

Solution. Let A alone fill the tank in t minutes,

then B alone will fill the tank in $(t-15)$ minutes

and C alone will fill the tank in $(t-15-5)$ *i.e.* $(t-20)$ minutes.

$\therefore$ According to given,

$$\frac{1}{t}+\frac{1}{t-15}=\frac{1}{t-20}$$

$$\Rightarrow \quad \frac{t-15+t}{t(t-15)}=\frac{1}{t-20}$$

$$\Rightarrow \quad (2t-15)(t-20)=t(t-15)$$

$\Rightarrow \quad 2t^2 - 55t + 300 = t^2 - 15t$

$\Rightarrow \quad t^2 - 40t + 300 = 0$

$\Rightarrow \quad (t - 10)(t - 30) = 0$

$\Rightarrow \quad t = 30, t = 10$ (neglecting) $(\because t > 20)$

Hence, the time required by pipe A to fill the tank = 30 minutes.

Example 13. *Two pipes can fill a tank in 20 minutes and 24 minutes respectively and a waste pipe can empty 3 gallons of water per minute. If all the three pipes working together can fill the tank in 15 minutes, find the capacity of the tank.*

Solution. Let the waste pipe take t minutes to empty the full tank.

According to given,

$$\frac{1}{20} + \frac{1}{24} - \frac{1}{t} = \frac{1}{15}$$

$$\Rightarrow \quad \frac{1}{t} = \frac{1}{20} + \frac{1}{24} - \frac{1}{15}$$

$$\Rightarrow \quad \frac{1}{t} = \frac{6 + 5 - 8}{120}$$

$$\Rightarrow \quad \frac{1}{t} = \frac{3}{120} \Rightarrow t = 40$$

$\because$ Volume of water emptied by the waste pipe in 1 minute = 3 gallons

So, volume of water emptied by the waste pipe in 40 minutes = $3 \times 40 = 120$ gallons.

$\therefore$ The capacity of the tank = 120 gallons.

Example 14. *Three pipes A, B and C can fill a tank together in 8 hours. After working at it together for 2 hours B is closed and A and C can fill the remaining part in 9 hours. Find the time in which B alone can fill the tank.*

Solution. Part of the tank filled by A, B and C in 1 hour = $\frac{1}{8}$.

$\therefore$ Part of the tank filled by A, B and C in 2 hours = $\frac{2}{8} = \frac{1}{4}$.

Remaining part = $1 - \frac{1}{4} = \frac{3}{4}$.

In 9 hours A and C can fill = $\frac{3}{4}$ part of the tank

$\therefore$ In 1 hour A and C can fill = $\frac{3}{4} \times \frac{1}{9} = \frac{1}{12}$ part of the tank.

Now, B's 1 hour work = (A + B + C)'s 1 hour work – (A + C)'s 1 hour work

$$= \frac{1}{8} - \frac{1}{12} = \frac{1}{24}$$

Hence, B alone can fill the tank in 24 hours.

EXERCISE 1.4

1. Two pipes A and B can fill the tank in 24 minutes and 36 minutes respectively. If both the pipes are opened together, then find the time in which the tank will be filled completely.
2. A pipe can fill a tank in 20 minutes and another pipe can empty the same tank in 60 minutes. If both the pipes are opened together, then find how much time will they take to fill the tank?

3. A pipe can fill a tank in 40 minutes. Due to a leakage in the bottom it took 60 minutes to fill the tank. How much time will it take for the leakage to empty the full tank?

4. A pump can fill a tank with water in 2 hours. Because of a leakage it took $2\frac{1}{3}$ hours to fill the tank. How much time will it take for the leakage to drain all the water of the full tank?

5. Two pipes A and B running together can fill a tank in 6 minutes. If pipe A takes 5 minutes less than B to fill the tank, find the time taken by pipe B to fill the tank alone.

6. Pipe A can fill the tank 3 times faster than pipe B. If both pipes A and B can fill the tank in 36 minutes running together, find how much time will pipe A take to fill the tank alone?

7. Two pipes A and B can fill a cistern in 32 minutes and 48 minutes respectively. Both the pipes are opened together for some time and then pipe B is turned off. Find when pipe B must be turned off so that the cistern gets filled in 24 minutes?

8. Two pipes A and B can fill a tank in 20 hours and 16 hours respectively. Pipe B alone is kept open for $\frac{1}{4}$th of time and both pipes are kept open for the remaining time. Find the time required to fill the tank.

9. Two pipes A and B can fill a tank in $\frac{1}{2}$ hour and 1 hour respectively. A pipe C can empty the tank in 2 hours. If all three pipes are opened simultaneously, how long does it take to fill the empty tank?

10. Two pipes A and B can fill a tank in 20 minutes and 60 minutes respectively. There is an outlet pipe C at the bottom of the tank. If all the three pipes are opened together it took 40 minutes to fill the tank. Find the time taken by outlet C to empty the full tank working alone.

11. Three pipes A, B and C can fill a tank in 72 minutes. If all the three pipes remain opened for 36 minutes and then pipe C is closed it took 1 hour more to fill the tank by pipes A and B. Find the time required to fill the tank by pipe C alone.

12. Two pipes P and Q can fill a tank in 60 minutes and 30 minutes respectively. Third pipe R drains 5 litres of water per minute. If all the three pipes are opened it took 2 hours to fill the tank. Find the capacity of the tank.

13. A tank is filled by three pipes with uniform flow. The first two pipes operating simultaneously fill the tank in the same time during which the tank is filled by third pipe alone. The second pipe fills the tank 5 hours faster than the first pipe and 4 hours slower than the third pipe. Find the time required by first pipe to fill the tank alone.

14. Three pipes A, B and C can fill a tank in 5 hours working simultaneously. The pipe C is twice as faster B and B is twice as fast as A. How much time will pipe A alone take to full the tank?

15. Two taps A and B can fill a tank in 4 and 6 hours respectively. The pipes are opened simultaneously and it takes 6 minutes more to fill the tank due to leakage. If the tank is full, then find the time taken by the leakage to empty the tank.

16. A tank can be filled by two inlet pipes P and Q in 15 minutes and 20 minutes respectively. Another outlet pipe R can empty the full tank in 24 minutes. If outlet pipe R is opened 6 minutes after the pipes P and Q are opened, find the total time to fill the tank.

17. Three pipes A, B and C can fill a tank together in 2 hours. Pipes B and C take 3 hours to fill the tank, while pipes A and C take 4 hours to fill the tank. Find the time taken by each pipe to fill the tank alone.

Answers

1. 14 min 24 sec
2. 30 minutes
3. 2 hours
4. 14 hours
5. 15 minutes
6. 48 minutes
7. 12 minutes
8. 10 hours
9. 24 minutes
10. 24 minutes
11. 3 hours
12. 120 litres

13. 15 hours **14.** 35 hours **15.** 60 hours **16.** 10 minutes

17. Pipe A: 6 hours, Pipe B: 4 hours, Pipe C: 12 hours